

Nonequilibrium Photons as a Signature of Quark-hadron Phase Transition

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(December 1998)

Abstract

We study the nonequilibrium photon production in the quark-hadron phase transition, using the Friedberg-Lee type solitons as a working model for quark-hadron physics. We propose that to search for nonequilibrium photons in the direct photon measurements of heavy-ion collisions may be a characteristic test of the transition from the quark-gluon to hadronic phases.

PACS numbers: 12.38.Mh, 25.75.-q, 11.15.Tk

Typeset using REVTeX

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Quark-confinement is an unresolved problem in QCD physics. Nevertheless, it is generally believed that quarks are deconfined at high energies. One of the primary goals of relativistic heavy-ion-collision experiments such as BNL-AGS and CERN-SPS is to create a hot central region and a dense fragmentation region in which quark-gluon plasma (QGP) may be formed. One of the ways for testing the occurrence of QGP is to observe the spectrum of the emitted photons. An advantage of measuring the direct photons is that photons do not suffer from strong final-state interactions as hadrons do, so they can be used to monitor the initial stages of the colliding heavy ions [1].

In relativistic heavy-ion collisions, a large number of π^0 and η are produced by soft QCD processes. Their two-photon decays constitute the major source of photon emission. Other photon sources arise from radiative decays of other mesons and baryon resonances, as well as hadron scattering processes. Also, if QGP is formed, photons will be produced via quark-antiquark annihilations or quark-gluon Compton scatterings. It is an experimental challenge to separate out the photon yield into a part arising from the decays of produced π^0 and η and the "single photon" part from other sources. For instance, the part from decays of mesons can be subtracted by reconstructing the distribution of the two-photon invariant mass of all photon pairs. If this can be done, then the excess single photon spectrum can be a probe of the properties of the nuclear matter or QGP, or even a discriminator between the two phases. However, recent studies [2–4] have shown that the two single photon spectra are similar, and distinguishing between them might need further high-precision direct photon measurements or other complementary measurements such as dilepton production and J/ψ suppression.

In this Letter, we give an attempt to utilize the nonequilibrium nature of the photon emission associated with the quark-hadron phase transition (QHPT) to test the formation of QGP. In essence, we calculate the particle production from the release of latent heat of the phase transition, adopting the formalism of nonequilibrium field theory that is well-suited to study the dynamics of nonequilibrium processes [5–7]. To model the QHPT, we use the Friedberg-Lee (FL) phenomenological nontopological soliton model [8], which is simple and

adequate enough for our present considerations. In fact, FL model has been applied to fit the hadronic properties [9], as well as to discuss the QHPT of the early Universe [10,11]. We believe that the particle production considered here is generic to all Landau-Ginzberg type models of QHPT. Our main result will be the photon spectrum produced from the QHPT, and the experimental signature will be briefly discussed.

We begin with the simplest FL model whose Lagrangian is given by [8]

$$\mathcal{L}_\sigma = i \sum_{i=1}^{n_F} \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \sum_{i=1}^{n_F} f \sigma \bar{\psi}_i \psi_i, \quad (1)$$

where ψ_i represents the quark field, $n_F = 2$ is the number of light flavors, σ is a real scalar field, f is a coupling constant, and the potential $U(\sigma)$ is given by

$$U(\sigma) = \frac{a}{2!} \sigma^2 - \frac{b}{3!} \sigma^3 + \frac{c}{4!} \sigma^4 + B, \quad (2)$$

where a , b , and c are positive parameters, and B is the bag constant. Note that $U(\sigma)$ has a local minimum at $\sigma = 0$, and an absolute minimum at $\sigma = \sigma_c$ which denotes the true vacuum with $U(\sigma_c) = 0$, separated by a potential barrier. A typical potential is shown in Fig. 1. Due to the nonlinearity of $U(\sigma)$, the model bears nontopological soliton solutions which are identified with hadrons. Essentially, inside a hadron is a perturbative vacuum with $\sigma \simeq 0$ decorated with localized free quarks, whereas outside is the true vacuum.

The simple picture of the production of nonequilibrium photons from the QHPT is the following. Suppose that a thermalized QGP is produced initially in a relativistic nucleus-nucleus collision, which has a perturbative vacuum with $\sigma = 0$. It then expands and cools, and undergoes hadronization in a phase transition. Here we assume that the transition is weakly first-order as consistent with lattice QCD calculations [12]. As such, transition to a state with $\sigma \simeq \phi_i$ (see Fig. 1) by quantum tunneling (or thermal activation) will occur through nucleation of hadronic bubbles in the QGP. Due to the plasma expansion, it is conceivable that the σ field inside a bubble might get trapped at $\sigma \simeq \phi_i$ as the interaction rate of σ is small compared to expansion rate. When the expansion slows down, σ will start to oscillate and relax via production of the particles, to which the σ particle is coupled,

to the true vacuum. This nonequilibrium stage will take place for a time-scale of about 10 fm, as revealed by hydrodynamical simulations (for example, see Ref. [3] and references therein). Typically, during this stage the dominant channel for the relaxation is the production of strong interacting particles such as σ quanta, gluons and other mesons. These particles will be immediately trapped and thermalized in the nuclear matter. Perhaps the σ quanta are short-lived to decay into photons before thermalization. If this happens, the decay photons would be a source for nonequilibrium photons. However, our current interest is the direct photon production driven by the oscillations of the σ field due to parametric amplification. We will ignore the hydrodynamic expansion and simplify the dynamics by adopting a "quenched" phase transition from an initial state in local thermodynamic equilibrium at a temperature slightly above the critical temperature cooled instantaneously to zero temperature. This "quench" approximation is far from a definitive description of the dynamics. Nevertheless, it captures the qualitative features and allows a simple but concrete calculation [6,7].

At tree level the σ field is inert to electromagnetic interaction, but it can couple to photon through a quark loop. After integrating out the quark field, we obtain an one-loop effective Lagrangian including strong interacting final states,

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}m_V^2 B_\mu B^\mu \\ & - \frac{g}{4\sigma_c}(\sigma - \sigma_c)F_{\mu\nu}F^{\mu\nu} - \frac{h}{4\sigma_c}(\sigma - \sigma_c)G_{\mu\nu}F^{\mu\nu}, \end{aligned} \quad (3)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the photon, and $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is a quark-antiquark vector meson with mass m_V . It is straightforward to calculate the coupling constant $g \simeq 2.6 \times 10^{-3}$. But the coupling constant h would depend on the wave function at the origin of the vector meson, $\Psi_V(0)$. Assuming $m_V \simeq 2$ GeV and taking a conservative value for $|\Psi_V(0)|^2 \simeq 0.1 \text{ fm}^{-3}$ [9], we find $h \simeq 1.8 \times 10^{-2}$. Note that effective strong interacting vertices such as σgg (where g denotes a gluon) and $\sigma\pi\pi$ should also appear at one-loop level. However, we have omitted them in the effective Lagrangian (3) because their effects to the particle production can be fully represented by the σ strong self-couplings. Since we

are concerned with photon production only, we integrate out the vector meson and obtain

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g}{4\sigma_c}(\sigma - \sigma_c)F_{\mu\nu}F^{\mu\nu} - \frac{h^2}{8m_V^2\sigma_c^2}\partial_\mu\sigma\partial_\alpha\sigma F^{\mu\nu}F^\alpha{}_\nu, \quad (4)$$

where we have dropped off higher derivative terms. At this stage, we should emphasize that the above obtained effective Lagrangian for the processes such as $\sigma' \rightarrow 2\gamma$ as well as $2\sigma' \rightarrow 2\gamma$ (here σ' is the shifted field with $\sigma = \sigma' + \sigma_c$) is clearly understood in perturbation theory. It is not certain whether this effective Lagrangian can also describe the nonequilibrium situation. In other words, the effective vertices that account for the above mentioned processes may be modified in the strongly out-of-equilibrium situation. The fuller study of off-equilibrium effective vertices by integrating out the quark fields and vector meson is a challenging task that lies beyond the scope of this paper, but certainly deserves to be taken up in the near future. However, in this work, we will use this effective Lagrangian (4) to compute the nonequilibrium photon production.

Following the nonequilibrium closed time path formalism [5–7], the nonequilibrium effective Lagrangian is given by

$$\mathcal{L}_{neq} = \mathcal{L}[\sigma^+, A_\mu^+] - \mathcal{L}[\sigma^-, A_\mu^-], \quad (5)$$

where $+$ ($-$) denotes the forward (backward) time branches. We then split σ^\pm into a mean field and the quantum fluctuating fields:

$$\sigma^\pm(\vec{x}, t) = \phi(t) + \chi^\pm(\vec{x}, t), \quad (6)$$

with the tadpole conditions,

$$\langle \chi^\pm(\vec{x}, t) \rangle = 0. \quad (7)$$

This tadpole conditions will be imposed to all orders in the corresponding expansion to obtain the nonequilibrium equations of motion from the first principles approach.

To derive the nonequilibrium evolution equations that consistently take into account quantum fluctuation effects from the strong σ self-interaction, we adopt the following Hartree factorization for χ implemented for both \pm components [5–7]:

$$\begin{aligned}\chi^4 &\rightarrow 6\langle\chi^2\rangle\chi^2 + \text{constant}, \\ \chi^3 &\rightarrow 3\langle\chi^2\rangle\chi.\end{aligned}\tag{8}$$

The expectation value will be determined self-consistently. Before proceeding any further, it is worth noting that although the above Hartree factorization is uncontrolled in this effective field theory that involves a single scalar field [5–7], our justification of using this approximation is based on the fact that it provides a non-perturbative framework that allows us to treat the strong σ dynamics self-consistently.

After doing this factorization, the Lagrangian then becomes

$$\begin{aligned}\mathcal{L}[\phi(t) + \chi^+, A_\mu^+] - \mathcal{L}[\phi(t) + \chi^-, A_\mu^-] &= \left\{ \frac{1}{2}(\partial\chi^+)^2 - U(t)\chi^+ - \frac{1}{2}M_\chi^2(t)\chi^{+2} - \frac{1}{4}F_{\mu\nu}^+ F^{+\mu\nu} \right. \\ &\quad - \frac{g}{4\sigma_c}(\phi(t) - \sigma_c)F_{\mu\nu}^+ F^{+\mu\nu} - \frac{g}{4\sigma_c}\chi^+ F^{+\mu\nu} F_{\mu\nu}^+ - \frac{h^2}{8m_V^2\sigma_c^2}\dot{\phi}^2(t)F^{+0i}F^{+0}_i \\ &\quad \left. - \frac{h^2}{4m_V^2\sigma_c^2}\dot{\phi}(t)\partial_\alpha\chi^+ F^{+0i}F^{+\alpha}_i - \frac{h^2}{8m_V^2\sigma_c^2}\partial_\mu\chi^+\partial_\alpha\chi^+ F^{+\mu\nu}F^{+\alpha}_\nu \right\} - \{+ \rightarrow -\},\end{aligned}\tag{9}$$

where

$$\begin{aligned}U(t) &= \ddot{\phi}(t) + \left[a - \frac{b}{2}\phi(t) + \frac{c}{6}\phi^2(t) + \frac{c}{2}\Sigma(t) \right] \phi(t) - \frac{b}{2}\langle\chi^2\rangle(t), \\ M_\chi^2(t) &= a - b\phi(t) + \frac{c}{2}\phi^2(t) + \frac{c}{2}\Sigma(t), \\ \Sigma(t) &= \langle\chi^2\rangle(t) - \langle\chi^2\rangle(0).\end{aligned}\tag{10}$$

Here, we have performed a subtraction of $\langle\chi^2\rangle(t)$ at $t = 0$ absorbing $\langle\chi^2\rangle(0)$ into the finite renormalization of a . Besides, since we are interested in the processes of direct photon production driven by a time dependent $\phi(t)$ field, in which the photons do not appear in the intermediate states, to avoid the gauge ambiguities, we can work on the Coulomb gauge, and concentrate only on physical transverse gauge fields, \vec{A}_T [7].

With the above Hartree-factorized Lagrangian in the Coulomb gauge, we perform a perturbative expansion in the weak couplings g and h^2 . However, the strong σ dynamics is treated non-perturbatively [7]. Following from the tadpole conditions (7), we obtain the following full one-loop equation of motion of $\phi(t)$ given by

$$\ddot{\phi}(t) + \left[a - \frac{b}{2}\phi(t) + \frac{c}{6}\phi^2(t) + \frac{c}{2}\Sigma(t) \right] \phi(t) - \frac{g}{2\sigma_c} \langle |\partial_\mu \vec{A}_T|^2 \rangle(t) - \frac{h^2}{4m_V^2\sigma_c^2} \left[\ddot{\phi}(t) + \dot{\phi}(t)\frac{d}{dt} \right] \langle |\dot{\vec{A}}_T|^2 \rangle(t) = 0. \quad (11)$$

The Heisenberg field equations can be read off from the quadratic part of the Lagrangian in the form

$$\begin{aligned} & \left[\frac{d^2}{dt^2} - \vec{\nabla}^2 + M_\chi^2(t) \right] \chi(t) = 0, \\ & \left[1 + \frac{g}{\sigma_c}(\phi(t) - \sigma_c) + \frac{h^2}{4m_V^2\sigma_c^2}\dot{\phi}^2(t) \right] \ddot{\vec{A}}_T(t) + \left[\frac{g}{\sigma_c}\dot{\phi}(t) + \frac{h^2}{2m_V^2\sigma_c^2}\dot{\phi}(t)\ddot{\phi}(t) \right] \dot{\vec{A}}_T(t) - \left[1 + \frac{g}{\sigma_c}(\phi(t) - \sigma_c) \right] \vec{\nabla}^2 \vec{A}_T(t) = 0. \end{aligned} \quad (12)$$

Now we decompose the fields χ and \vec{A}_T into their Fourier mode functions $U_{\vec{k}}(t)$ and $V_{\lambda\vec{k}}(t)$ respectively,

$$\begin{aligned} \chi(\vec{x}, t) &= \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\chi\vec{k}}}} \left[a_{\vec{k}} U_{\vec{k}}(t) e^{i\vec{k}\cdot\vec{x}} + \text{h.c.} \right], \\ \vec{A}_T(\vec{x}, t) &= \sum_{\lambda=1,2} \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{A\vec{k}}}} \left[b_{\lambda\vec{k}} V_{\lambda\vec{k}}(t) e^{i\vec{k}\cdot\vec{x}} + \text{h.c.} \right], \end{aligned} \quad (13)$$

where $a_{\vec{k}}$ and $b_{\lambda\vec{k}}$ are destruction operators, and $\vec{\epsilon}_{\lambda\vec{k}}$ are linear polarization unit vectors. The frequencies $\omega_{\chi\vec{k}}$ and $\omega_{A\vec{k}}$ can be determined from the initial states and will be specified below. Then the mode equations are

$$\begin{aligned} & \left[\frac{d^2}{dt^2} + k^2 + M_\chi^2(t) \right] U_k(t) = 0, \\ & \left\{ \left[1 + \frac{g}{\sigma_c}(\phi(t) - \sigma_c) + \frac{h^2}{4m_V^2\sigma_c^2}\dot{\phi}^2(t) \right] \frac{d^2}{dt^2} + \left[\frac{g}{\sigma_c}\dot{\phi}(t) + \frac{h^2}{2m_V^2\sigma_c^2}\dot{\phi}(t)\ddot{\phi}(t) \right] \frac{d}{dt} + k^2 \left[1 + \frac{g}{\sigma_c}(\phi(t) - \sigma_c) \right] \right\} V_{\lambda k}(t) = 0, \end{aligned} \quad (14)$$

with the vacuum expectation values given by

$$\begin{aligned} \langle \chi^2 \rangle(t) &= \int^\Lambda \frac{d^3k}{2(2\pi)^3\omega_{\chi k}} |U_k(t)|^2, \\ \langle |\partial_\mu \vec{A}_T|^2 \rangle(t) &= \sum_\lambda \int^\Lambda \frac{d^3k}{2(2\pi)^3\omega_{A k}} \left[|\dot{V}_{\lambda k}(t)|^2 - k^2 |V_{\lambda k}(t)|^2 \right], \end{aligned} \quad (15)$$

where we set the cutoff scale $\Lambda \simeq m_V$, and the expectation value of the number operator for the asymptotic photons with momentum \vec{k} is given by [7]

$$\begin{aligned}\langle \mathbf{N}_k(t) \rangle &= \frac{1}{2k} \left[\dot{\vec{A}}_T(\vec{k}, t) \cdot \dot{\vec{A}}_T(-\vec{k}, t) + k^2 \vec{A}_T(\vec{k}, t) \cdot \vec{A}_T(-\vec{k}, t) \right] - 1 \\ &= \frac{1}{2k^2} \sum_{\lambda} \left[|\dot{V}_{\lambda k}(t)|^2 + k^2 |V_{\lambda k}(t)|^2 \right] - 1.\end{aligned}\tag{16}$$

This gives the spectral number density of the photons produced at time t , $dN(t)/d^3k$. To solve the evolution equations (11,14), we propose the following initial conditions for the mode functions at the time of "quench" [7]:

$$\begin{aligned}U_k(0) &= 1, \quad \dot{U}_k(0) = -i\omega_{\chi k}, \quad \omega_{\chi k}^2 = k^2 + a + b\phi_i + \frac{c}{2}\phi_i^2; \\ V_{\lambda k}(0) &= 1, \quad \dot{V}_{\lambda k}(0) = -i\omega_{Ak}, \quad \omega_{Ak} = k,\end{aligned}\tag{17}$$

where the initial mode functions are chosen to be at zero temperature inside the hadronic bubbles with the mean field $\phi(t)$ displaced initially away from the equilibrium position (i.e., $\phi(0) = \phi_i \neq 0$). The above specified initial conditions are physically plausible and simple enough for us to investigate a quantitative description of the dynamics.

Let us use the FL model parameters: $a = 1.6 \text{ fm}^{-2}$, $b = 69 \text{ fm}^{-1}$, $c = 500$, and $f = 9.57$. This implies that $\sigma_c = 0.36 \text{ fm}^{-1}$. This set of parameters has been used for fitting hadronic mass spectrum [9], and with this potential (2) (see Fig. 1) the phase transition is weakly first-order at finite temperature [10]. We choose the initial amplitude $\phi_i = 0.07 \text{ fm}^{-1}$ to solve Eq. (11).

In Fig. 2, the time evolution of the mean field $\phi(t)$ is shown. It can be seen that ϕ is oscillating about a mean value of about 1.1 fm^{-1} , which is significantly different from the classical value σ_c . Also, $\phi(t)$ oscillates with a frequency $\omega_{\phi} \simeq 8.8 \text{ fm}^{-1}$. This is due to the quantum fluctuation effects coming from the σ field and the gauge field \vec{A}_T to the motion of $\phi(t)$ (11). The back reactions of the quantum fields also account for the damping of $\phi(t)$ with time. In Fig. 3, we plot the time dependence of the photon number density integrated over momentum \vec{k} , $N(t)$. As expected, $N(t)$ oscillates with the same frequency as $\phi(t)$. Although photons are produced efficiently during the first half of an oscillation, almost all of them

are re-absorbed to the background during the second half of the oscillation. However, it is important to note that the time average of $N(t)$ is indeed increasing with time, that is to say, photons are effectively produced from ϕ oscillations due to parametric amplification [6].

It is useful to calculate the time-averaged invariant photon production rate, kdR/d^3k , where

$$dR = \frac{1}{T} \int_0^T \frac{dN(t)}{dt} dt, \quad (18)$$

over a period from the initial time to time T . The results are shown in Fig. 4 with $T = 3, 5, 10$ fm. Also shown are the thermal photon production rate from a quark-gluon plasma and a hadron gas taken from Ref. [13]. It is apparent that the ϕ oscillations with frequency ω_ϕ produce non-thermal photons whose spectrum peaks around two photon momenta, $k = \omega_\phi/2$ and $k = \omega_\phi$, which correspond to the unstable bands (or resonant bands) arising from the fact that the mode equations of \vec{A}_T (14) consistently depend on the time dependent mean field $\phi(t)$ [7]. Clearly, the photon production mechanism is that of parametric amplification. From the mode equations (14), we can easily recognize that the 4.4 fm^{-1} peak is resulted from the coupling σF^2 while the 8.8 fm^{-1} peak is from the interaction $(\partial\sigma)^2 F^2$. Note that the 4.4 fm^{-1} peak has a peak value of the production rate being comparable to that of thermal photons, while interestingly the 8.8 fm^{-1} peak is almost two orders of magnitude larger than the thermal photons. Although these results more or less depend on the choice of the FL model parameters, we do not find a change of two orders of magnitude. So these high-energy non-thermal photons can be a distinct signature of QGP formation.

In summary, we have computed the nonequilibrium photon production during the quark-hadron phase transition at high-temperature in the Friedberg-Lee model of quark-hadron physics. Under the "quench" approximation, the invariant production rate for nonequilibrium photons driven by the oscillation of the σ field due to parametric amplification is given, which is two orders of magnitude larger than that from a thermal quark-gluon plasma for photon energies around 2 GeV. These high-energy non-thermal photons can be a potential test of the formation of quark-gluon plasma in relativistic heavy-ion-collision experiments.

Of course, in order to compare the results with experimental data on direct photons, a more realistic dynamics of the phase transition should be considered and the nonequilibrium photon production rate should be convolved with the expansion of the plasma. This work is in progress.

We would like to thank D. Boyanovsky and S.-P. Li for their useful discussions. The work of D.S.L. (K.W.N.) was supported in part by the National Science Council, ROC under the Grant NSC88-2112-M-259-001 (NSC88-2112-M-001-042).

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FIGURE CAPTIONS

Fig. 1. Scalar potential $U(\sigma)$ with $a = 1.6 \text{ fm}^{-2}$, $b = 69 \text{ fm}^{-1}$, and $c = 500$, and ϕ_i is the initial value.

Fig. 2. Time evolution of the mean field $\phi(t)$.

Fig. 3. Time evolution of produced total number density of nonequilibrium photons from quark-hadron phase transition.

Fig. 4. Spectral production rate of nonequilibrium photons from quark-hadron phase transition, calculated from Eq. (18) with $T = 3, 5, 10 \text{ fm}$, drawn with solid lines. Dashed lines are the rates for hadron gas and quark-gluon plasma taken from Ref. [13].

FIGURES

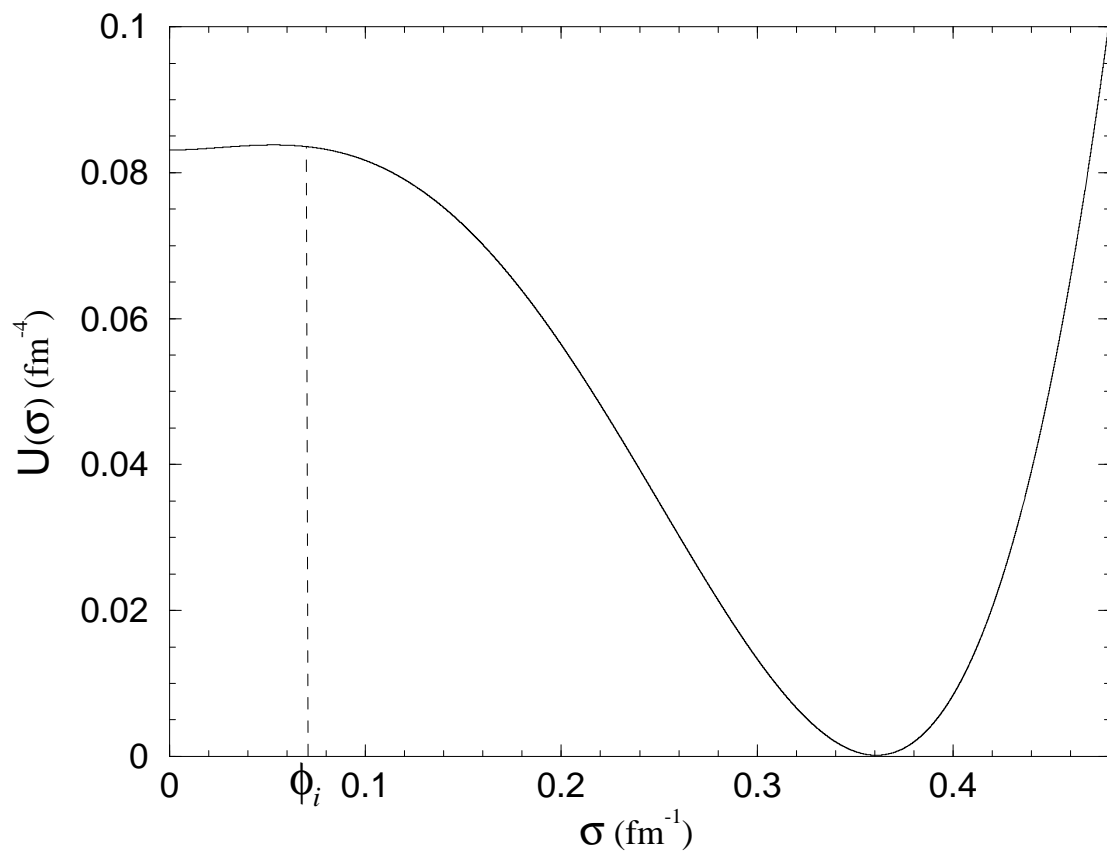


FIG. 1.

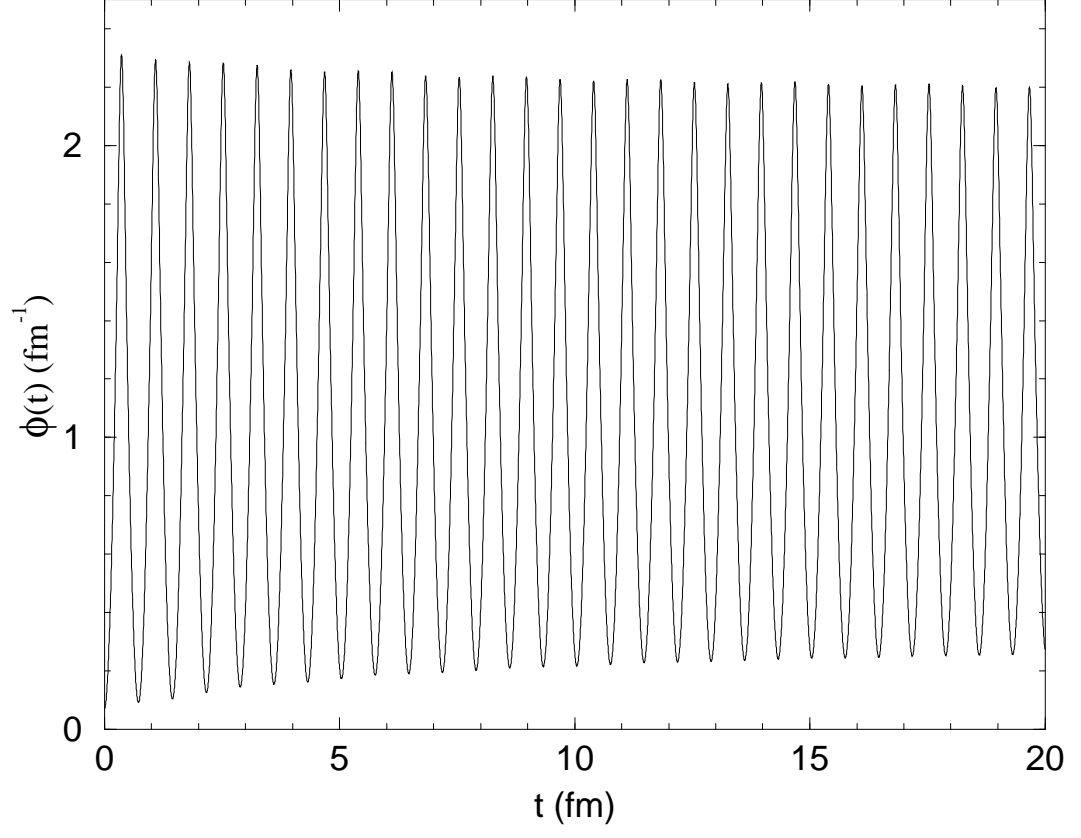


FIG. 2.

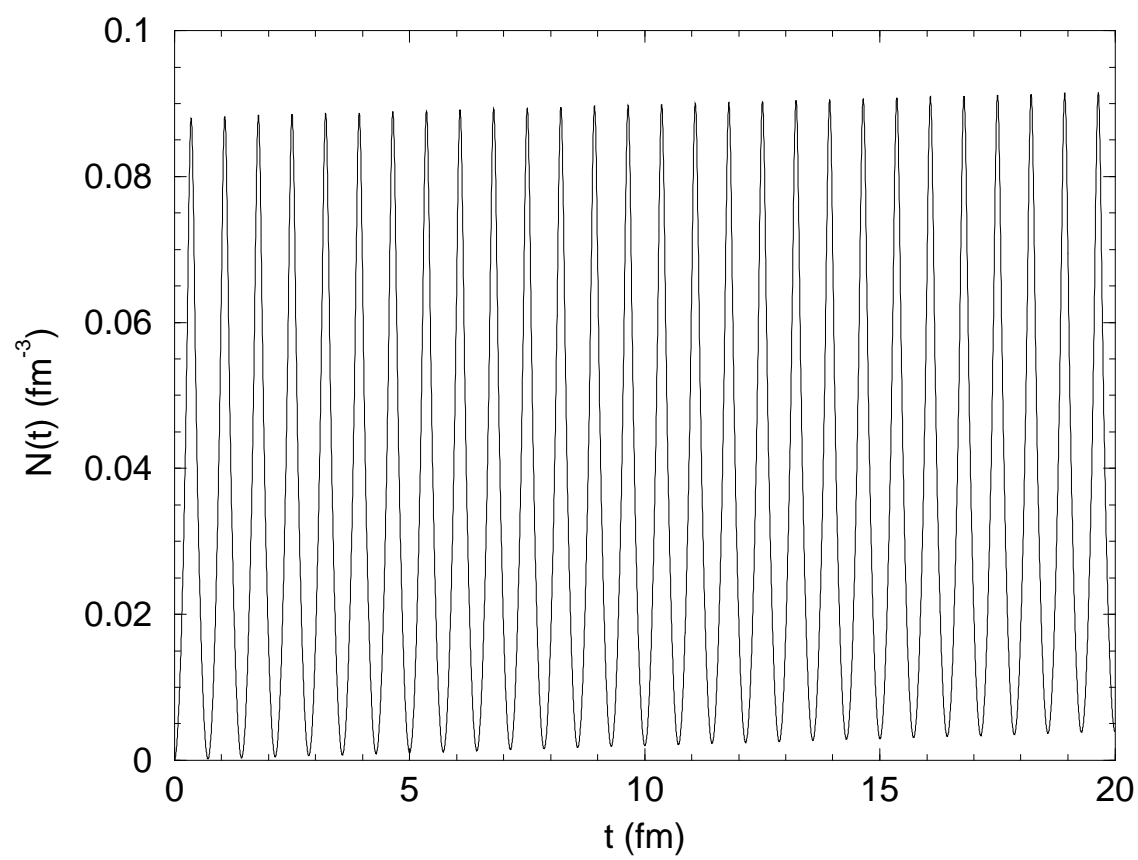


FIG. 3.

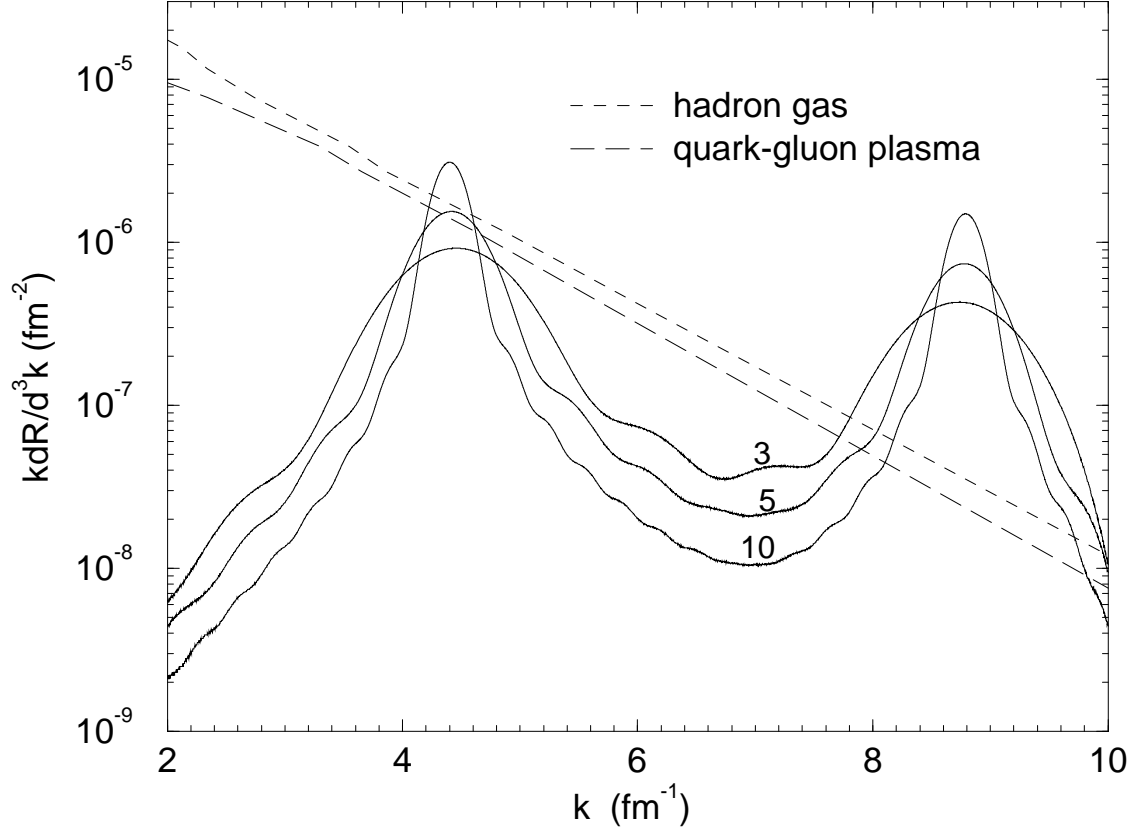


FIG. 4.